Binary Search Tree (BST) notes

Prof Bill - Feb 2020

The reading:

- Sedgewick Algos 3.2 BST, algs4.cs.princeton.edu/32bst
- Sedgewick Java 4.4 Symbol tables, introcs.cs.princeton.edu/java/44st/
- This **animation** is great: www.cs.usfca.edu/~galles/visualization/BST.html
- A nice lecture: www.cs.cmu.edu/~adamchik/15-121/lectures/Trees/trees.html

Sections are:

A. Binary trees

B. Binary search trees (BST)

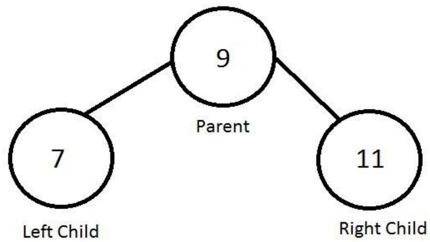
C. BST ADT

D. Terms

thanks...yow, bill

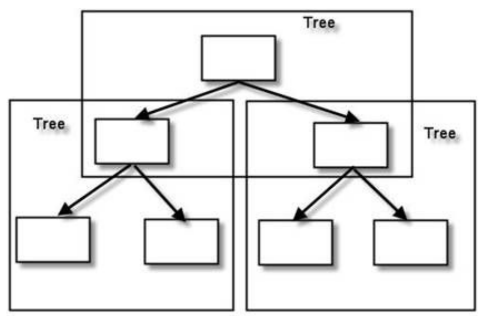
A. Binary Trees

Binary trees have nodes like a linked list. Each **node** has data (key, value) and then left and right node pointers. The **root** is the first node in the tree.



Source: cppbetterexplained.com/binary-search-trees/

Binary trees are **recursive structures** because nodes have nodes in them.
Also, subtrees behave just like the overall tree. Makes for easy recursive methods.



Binary tree consisting of 3 binary trees

Source: www.sqa.org.uk/e-learning/LinkedDS04CD/page_30.htm

Traversal:

- **Preorder**: root, left, right
- **Inorder**: left, root, right   // sorted order in a BST
- **Postorder**: left, right, root

/* memory helper: 1) root determines pre, in, or post and 2) left always before right */

Pseudocode… start process with call: inorder(root):

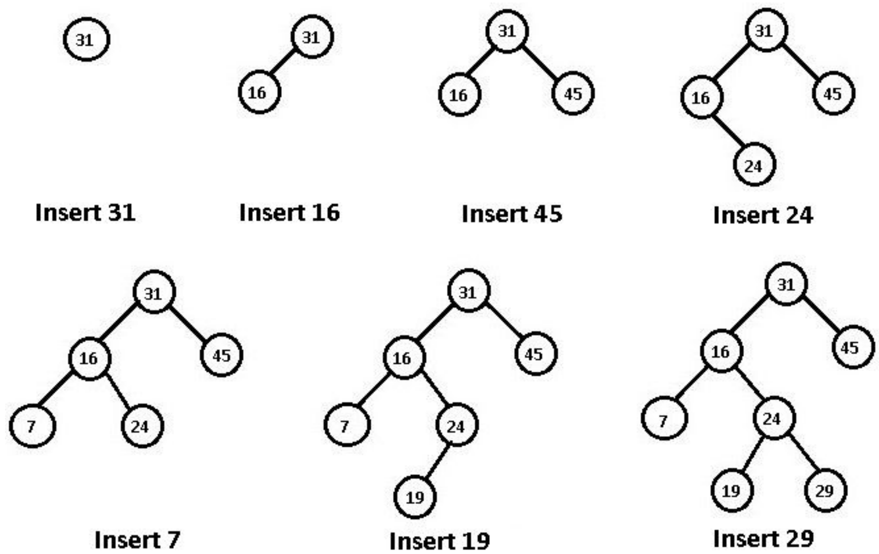
```
// inorder traversal to print binary tree
void inorder( Node n)
  if n == null then return
  inorder( n.left)
  print n
  inorder( n.right)
```

B. Binary Search Trees (BST)

A binary tree + this magic…**two rules** for every node:

1. left child is less than (<) node
2. right child is greater than (>) node

That's it. Let's build one. An empty BST is root = null (not shown below).
Below: the root is the first node added; in this case 31.



.

Source: csegeek.com/csegeek/view/tutorials/algorithms/trees/tree_part2.php

Notice in our example:

- The root doesn't change when adding to the tree
- Every new node is added as a leaf

Performance for BST magic,

➔ Average performance is O(log n), problem cut in half with each subtree
➔ Worst-case performance is O(n), an unbalanced tree turns into a linked list (dop!)

There are 3 important methods in the BST ADT:

1. put(K key, V value) - we just did this
2. V get(K key)
3. V remove(K key)

With **put()** - Often, we just show the keys. The value is there or it's just keys (like a set). Use the same left/right algorithm as get() below. New node is always a leaf!

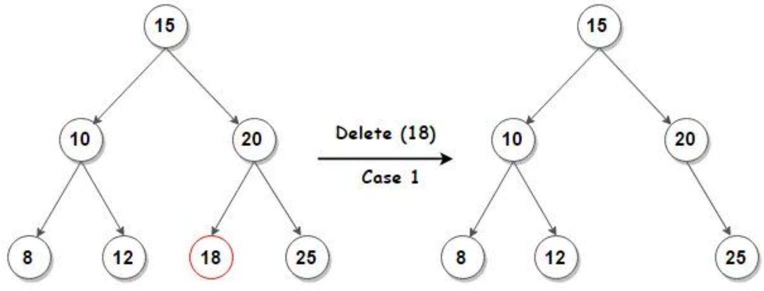
Here's **get()** pseudocode… it's a recursive search:

```
get( K key) {
   return getNode( root, key)              // start at root
}

V getNode( Node n, K key) {
   if n == null then return null           // NOT found

   if key == node.key return node.value    // FOUND

   if key < node.key
        return getNode( node.left, key) // look LEFT
   else
        return getNode( node.right, key)   // look RIGHT
}
```

Remove? Well, it's a little tougher.

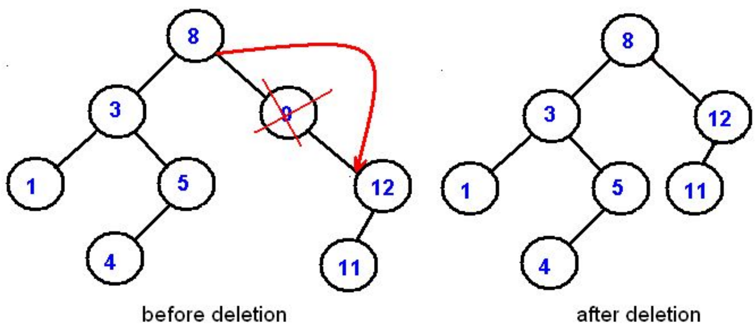
Three cases, removing a node with no children (leaf), one child, and two children:

➔ No children (leaf) - null out the parent's link to the node (easy)



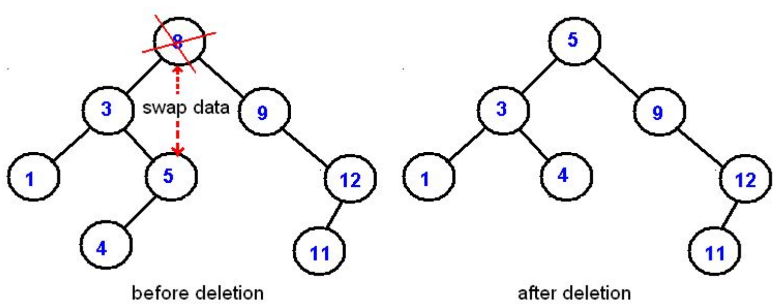
Source: www.techiedelight.com/deletion-from-bst/

➔ One child - replace node with its child (pretty easy)



Source: www.cs.cmu.edu/~adamchik/15-121/lectures/Trees/pix/del01.bmp

➔ Two children - replace node with predecessor (largest node in left subtree, tougher)



About two children - Successor is ok too, smallest node in right subtree; Pred/Succ are always a leaf or one-child node. Yes?

C. BST ADT

Binary Search Tree (BST) - All operations are average O(log n), worst case O(n).

The worst case performance happens when the BST becomes unbalanced, where one subtree is much larger (and longer) than another.

Methods: put(key,value), value get(key), value min(), value max(), print()

Pseudocode… each public method starts at the root and calls a corresponding private, recursive method that uses BST nodes:

```
BinarySearchTree
   Node {
      K, V (key, value)
      Node left
      Node right
   }
   private Node root;

// two put methods: public BST method and private recursive node method
put( K key, V value)
   n = new BST node
   if root == null
      then root = n
   else
      putNode( root, n)

private putNode( Node n, Node putNode)
   if putNode.key < n.key    // put in LEFT subtree
      if n.left == null
         n.left = putNode
      else
         putNode( n.left, putNode)
   else if putNode.key > n.key    // put in RIGHT subtree
      if n.right == null
         n.right = putNode
      else
         putNode( n.right, putNode)
```

```
// two get methods; get value for this key if found in BST
V get( K key)
    if root == null return null    // empty
    return getNode( root, key)

private V getNode( Node n, K key)
    if n == null, then return null    // not found
    if n.key == key
        return n.value    // FOUND - return it
    else if key < n.key
        return getNode( n.left, key)    // look LEFT
    else
        return getNode( n.right, key)    // look RIGHT

// two min methods; return the leftmost node, which is min
V min()
    if root == null, then return null
    return minNode( root)

private V minNode( Node n)
    if n.left == null
        return n.value    // no more left nodes, this is MIN
    else
        return minNode( n.left)    // go left again

// print BST keys in sorted order
print()
    printNode( root)

private printNode( Node n)
    if n == null, then return
    printNode( n.left)
    print n.data
    printNode( n.right)
```

Notes:

- All this recursion is **tail recursion** and can be easily replaced by iteration.

D. Terms

Binary tree and BST terms include:

node
root
parent
child
leaf

get, put, min, print
inorder, preorder, postorder

balanced
expected: O(log n), worst: O(n)

Java: Comparable
Java, TreeMap, www.baeldung.com/java-treemap